Nonlinear Aerodynamics of Bodies of Revolution in Free Flight

L. L. LEVY JR.* AND M. TOBAK*
NASA Ames Research Center, Moffett Field, Calif.

A nonlinear aerodynamic moment system is formulated for arbitrary motions of bodies of revolution in free flight. The total moment is shown to be compounded of the contributions from four simple motions, and a clear physical meaning is attached to each contribution. Particular distinction is made between the side-moment contribution caused by coning motion, and that as a result of classical Magnus motion. An example is included to illustrate a possible misinterpretation of results from motion analyses where this distinction is not made.

Nomenclature

C_D	= drag coefficient, 2 (drag force)/ρV^2S
C_L	= lift coefficient, 2 (lift force)/ρV^2S
C_Y	= side-force coefficient, 2 (side force)/ρV^2S
C_Z	= normal-force coefficient, 2 (normal force)/ρV^2S
C_l	= rolling-moment coefficient, $2\bar{L}/\rho V^2Sl$
C_m	= pitching-moment coefficient, $2\bar{M}/\rho V^2Sl$
C_n	= side-moment coefficient, $2\bar{N}/\rho V^2Sl$
$G[\delta(\xi),\dot{\psi}(\xi),\dot{\lambda}(\xi),q(\xi),r(\xi)]$	= functional notation: value at $\xi = t$ of a time-dependent function which depends on all values taken by the five argument functions $\delta(\xi)$, $\dot{\psi}(\xi)$, $\dot{\lambda}(\xi)$, $q(\xi)$, and $r(\xi)$ over the interval $0 \leq \xi \leq t$
I_{x_B},I_{y_B},I_{z_B}	= moments of inertia about body axes x_B,y_B,z_B; $I_{y_B} = I_{z_B}$
$\bar{L}$	= moment about axis of cylindrical symmetry (Fig. 1)
l	= reference length
$\bar{M}$	= moment about an axis normal to the plane of the resultant angle of attack (about y, Fig. 1)
m	= body mass
$\bar{N}$	= moment about an axis in the plane of the resultant angle of attack (about z, Fig. 1)
p_B,q_B,r_B	= components along the x_B,y_B,z_B axes, respectively, of the total angular velocity of the body axes relative to inertial space
q,r	= components of angular velocity along the y,z axes, respectively, [Eq. (4)]
S	= reference area
s	= dimensionless distance, $(1/l)\int Vdt$
t	= time
u_B,v_B,w_B	= components of flight velocity along x_B,y_B,z_B axes, respectively, (Fig. 1)
V	= flight velocity
x_B,y_B,z_B	= body-fixed axes, origin at center of gravity, x_B coincident with axis of cylindrical symmetry (Fig. 1)
x_B,y,z	= aerodynamic-moment axes, origin at center of gravity, x_B, z in the plane of the resultant angle of attack, y,z in the crossflow plane normal to the resultant angle-of-attack plane (Fig. 1)
$x_B,\tilde{y},\tilde{z}$	= nonrolling axes (with respect to inertial space), origin at center of gravity, $\tilde{y},\tilde{z}$ in the crossflow plane (Fig. 1)
γ	= dimensionless axial component of velocity [Fig. 1 and Eq. (2)]
δ	= magnitude of the dimensionless crossflow velocity in the aerodynamic-moment axis system [Fig. 1 and Eq. (2)]
ϵ	= $\tan\sigma$ [Fig. 1 and Eq. (2)]
λ	= angular inclination from the $\tilde{y}$ axis of the crossflow-velocity vector (Fig. 1)
$\tilde{\xi}$	= dimensionless complex-crossflow-velocity vector in the nonrolling axis system [Eq. (12)]
ρ	= atmospheric mass density
σ	= resultant angle of attack defined by x_B axis and velocity vector (Fig. 1)
$\dot{\phi}$	= coning rate of x_B axis about the constant-velocity vector of a body in level flight
$\tilde{\phi}$	= angular inclination from the $\tilde{y}$ axis of the y_B axis [Fig. 1 and Eq. (1)]
ψ	= angular inclination from the crossflow-velocity vector of the z_B axis (Fig. 1)
$(\dot{\ })$	= $(d/dt)(\ )$
$(\ )'$	= $(d/ds)(\ )$

Presented as Paper 70–205 at the AIAA 8th Aersospace Sciences Meeting, New York, N. Y., January 19–21, 1970; submitted March 2, 1970 and revision received June 8, 1970.
* Research Scientist. Member AIAA.

Introduction

AN earlier study[1] proposed a formulation of the aerodynamic moment system, which does not depend on a linearity assumption and is capable of accounting for the coupling between motions that may occur during large-amplitude nonplanar maneuvers. By nonlinear functional analysis, it was shown that the moments for an arbitrary motion about the center of gravity may be compounded of the contributions from four simple motions, three of which are well known. The fourth, coning motion, was investigated experimentally and shown to yield a side moment that was a potential causative agent in the occurrence of large amplitude circular limit motions.

The previous analysis was limited to cases in which the center of gravity traverses a straight and level path at constant velocity. The purpose of the present work is to extend the analysis to the general case where the motion of the center of gravity is arbitrary. It will be shown that, despite the greater freedom of motion, the moments still may be compounded of the contributions from the same four simple motions. Some of the consequences of the new formulation will be examined. In particular, it will be demonstrated that unless the contributions to side moment caused by coning and by Magnus motion are properly distinguished, programs for extracting nonlinear aerodynamic coefficients from flight data may yield misleading results.

Analysis

Nonlinear Formulation of Aerodynamic Moment System

The first and principal step in formulating the aerodynamic moment system, by the use of functional analysis, is the designation of the variables on which the moments depend. For the body of revolution, this can be done by considering the flow in a plane normal to the body axis of cylindrical symmetry (i.e., the crossflow plane) referred to a nonrolling axis system.

Crossflow plane

Let us attach body-axis coordinates x_B, y_B, and z_B to the center of gravity where x_B is alined with the axis of cylindrical symmetry. The plane formed by y_B, z_B is the crossflow plane, illustrated in Fig. 1a. We specify that the axis system x_B, $\tilde{y}$, and $\tilde{z}$ be nonrolling with respect to inertial space, so that the angle $\tilde{\phi}$ through which the body axes have rolled at any time t can be defined relative to the nonrolling axis system as

$$\tilde{\phi} = \int_0^t p_B d\tau \tag{1}$$

p_B being the body roll rate, that is, the component of angular velocity along the x_B axis. The velocity vector with magnitude δ is the projection of the resultant angle of attack in the crossflow plane. We have, from Fig. 1b

$$\begin{gathered} \delta = [(v_B/V)^2 + (w_B/V)^2]^{1/2} = \sin\sigma \\ \gamma = u_B/V = \cos\sigma,\ \epsilon = \delta/\gamma = \tan\sigma \end{gathered} \tag{2}$$

The angular inclination λ of the crossflow-velocity vector is measured relative to the nonrolling axis system; the angular rate $\dot{\lambda}$ is representative of the coning motion of the body about the velocity vector. We designate by ψ the angular inclination of the body axes from the crossflow-velocity vector; $\dot{\psi}$ is the angular rate which properly should characterize the motion producing classical Magnus-type moments. Clearly, the total roll rate p_B is the sum

$$p_B = \dot{\lambda} + \dot{\psi} \tag{3}$$

In addition, we have the components of angular velocity q_B, r_B, respectively, around the body axes y_B, z_B.

Functional dependence

It will be convenient to define forces and moments in the plane of and normal to the plane of the resultant angle of attack. To this end, we specify crossflow-plane coordinates y, z, with z being alined with the direction of δ. The moment about the y axis (i.e., about an axis normal to the plane of the resultant angle of attack) will be called the pitching moment $\bar{M}$. The moment about the z axis (i.e., about an axis in the plane of the resultant angle of attack) will be called the side moment $\bar{N}$. The moment about x_B will be called the rolling moment $\bar{L}$. We refer the angular velocities q_B, r_B of the body axes y_B, z_B to the axes y, z through the relation

$$q + ir = e^{i\psi}(q_B + ir_B) \tag{4}$$

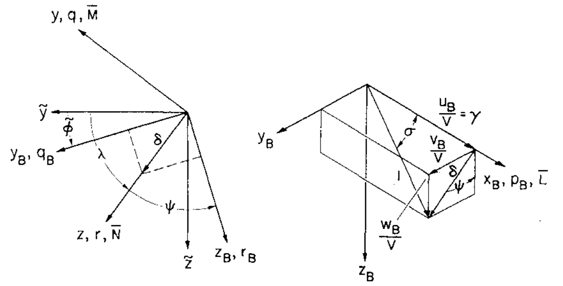
We now assume that the moment coefficients C_l, C_m, and C_n will be functionals of the variables δ, q, and r, and any two of p_B, $\dot{\lambda}$, and $\dot{\psi}$. We shall choose $\dot{\lambda}$ and $\dot{\psi}$ in order to bring out clearly the differing roles of coning and Magnus-type motions. Thus, for example, we specify that the pitching-moment coefficient C_m be a functional of the form

$$C_m(t) = G[\delta(\xi),\dot{\psi}(\xi),\dot{\lambda}(\xi),q(\xi),r(\xi)] \tag{5}$$

Approximate formulation

With the functional dependence specified, the formulation of indicial responses and an integral representation of $C_m(t)$ parallels that described in Ref. 1. Similarly, by assuming that although δ can be large, the rates $\dot{\psi}$, $\dot{\lambda}$, q, and r will be small, the integral form can be expanded about $\dot{\psi} = 0$, $\dot{\lambda} = 0$, $q = 0$, $r = 0$ to yield, to first order in the rates, a sum of stability derivatives. The result for $C_m(t)$ is

$$\begin{aligned} C_m(t) = {} & C_m(\infty;\delta(t)) + (\dot{\psi}l/V)(\infty;\delta(t)) + \\ & (\dot{\lambda}l/V)C_{m_{\dot{\lambda}}}(\infty;\delta(t)) + (ql/V)C_{m_q}(\infty;\delta(t)) + \\ & (rl/V)C_{m_r}(\infty;\delta(t)) + (\dot{\delta}l/V)C_{m_{\dot{\delta}}}(\delta(t)) \end{aligned} \tag{6}$$



a) CROSSFLOW PLANE b) RESULTANT ANGLE-OF-ATTACK PLANE

Fig. 1 Axes, angles, and velocity components in the crossflow and resultant angle-of-attack planes.

where, in the functional dependence notation, the infinity symbol indicates steady flow and, for brevity, the zeroes belonging to $\dot{\psi}$, $\dot{\lambda}$, q, and r have been omitted. Thus, the moment system is fully nonlinear in its dependence upon the resultant angle of attack and linear in its dependence upon the angular rates. Analogous expressions for $C_n(t)$ and $C_l(t)$ are obtained by substituting C_n or C_l wherever C_m appears in Eq. (6). Likewise, expressions for the normal-force coefficient $C_Z(t)$ and the side-force coefficient $C_Y(t)$ are obtained by substituting C_Z or C_Y for C_m.

Interpretation of results

Although it is possible to assign a precise physical meaning to each term in Eq. (6), it will be found more relevant to assign meanings to certain combinations of terms that appear together naturally in the equations of motion.

Let

$$C_{m_{\dot{\sigma}}} = C_{m_q} + \gamma C_{m_{\dot{\delta}}},\ C_{m_{\dot{\phi}}} = \gamma C_{m_{\dot{\lambda}}} + \delta C_{m_r} \tag{7}$$

and rewrite Eq. (6) as

$$\begin{aligned} C_m(t) = {} & C_m(\infty;\delta(t)) + \frac{\dot{\psi}l}{V} C_{m_{\dot{\psi}}}(\infty;\delta(t)) + \frac{\dot{\sigma}l}{V} C_{m_{\dot{\sigma}}}(\delta(t)) + \\ & \frac{\dot{\lambda}l}{V}\frac{1}{\gamma} C_{m_{\dot{\phi}}}(\infty;\delta(t)) + (q - \dot{\sigma})\frac{l}{V} C_{m_q}(\infty;\delta(t)) + \\ & (r - \epsilon\dot{\lambda})\frac{l}{V} C_{m_r}(\infty;\delta(t)) \end{aligned} \tag{8}$$

Now consider the case of no plunging. We have from Ref. 1

$$q = \dot{\sigma},\ r = \delta\dot{\phi},\ \dot{\lambda} = \gamma\dot{\phi} \tag{9}$$

where $\dot{\phi}$ is the coning rate. Then the last two terms in Eq. (8) vanish and the result is identical in form to Eq. (6) of Ref. 1. Hence, by direct comparison, the combination of terms $C_{m_q} + \gamma C_{m_{\dot{\delta}}}$ is the damping-in-pitch in planar motion ($\dot{\phi} = 0$, $\dot{\psi} = 0$) measured at a fixed inclination $\delta = \text{const}$. The combination of terms $\gamma C_{m_{\dot{\lambda}}} + \delta C_{m_r}$ is the rate of change with $\dot{\phi}l/V$, evaluated at $\dot{\phi} = 0$, of the pitching moment that would be measured in a steady coning motion $\delta = \text{const}$, $\dot{\phi} = \text{const}$, $\dot{\psi} = 0$. As before, the term $C_m(\infty;\delta(t))$ is the pitching moment caused by angle of attack in steady planar motion and the term $C_{m_{\dot{\psi}}}(\infty;\delta(t))$ is the rate of change with $\dot{\psi}l/V$, evaluated at $\dot{\psi} = 0$, of the pitching moment that would be measured in the classical Magnus experiment $\delta = \text{const}$, $\dot{\psi} = \text{const}$, and $\dot{\phi} = 0$.

For the general case, we return to Eq. (8), where now $q - \dot{\sigma}$ and $r - \epsilon\dot{\lambda}$ are not identically zero. However, it can be shown that retaining terms multiplied by these quantities in the equations of motion amounts to retaining products of stability derivatives, the magnitudes of which, under normal circumstances, are negligibly small compared to the principal contributions. Therefore, it is justifiable

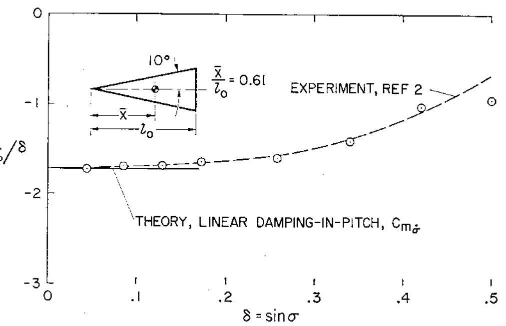


Fig. 2 Nonlinear side-moment coefficient caused by coning for a 10° cone at Mach number 2.

to discard the terms multiplied by $q - \dot\sigma$ and $r - \epsilon\dot\lambda$ in the general case as well. This is a substantial simplification, since there is then no necessity to undertake the difficult separate experiments for C_{m_q} alone and C_{m_r} alone that would be required were $q - \dot\sigma$ and $r - \epsilon\dot\lambda$ significantly different from zero. Thus, just as in the nonplunging case, the moments due to an arbitrary motion may be compounded of the contributions from four simple motions: steady angle of attack, planar damping-in-pitch at constant angle of attack, spin at constant angle of attack, and coning at constant angle of attack. Of the four motions, three have been studied extensively. The fourth, coning motion, only recently has been the subject of experimental and theoretical investigation,[2,3] and has been shown capable of making a significant nonlinear contribution to the side moment.

Further simplifications to the moment system

As in the nonplunging case, Eq. (8) and the analogous expressions for C_n and C_l admit the existence of moment contributions that normally may be eliminated on the basis of symmetry arguments. The contributions $C_{m_{\dot\phi}}(\infty;\delta(t))$, $C_{m_{\dot\psi}}(\infty;\delta(t))$, $C_n(\infty;\delta(t))$, and $C_{n_{\dot\sigma}}(\delta(t))$ may be eliminated on this basis. Further, none of the rolling-moment contributions would appear to be significant. We believe that in certain cases[2] the classical Magnus term $C_{n_{\dot\psi}}$ also may be eliminated. However, we shall retain it in order to point out a difficulty that arises *if* $C_{n_{\dot\phi}}$ and $C_{n_{\dot\psi}}$ are not properly distinguished. Under these conditions, the reduced moment system may be written

$$\begin{aligned} C_m(t) &= C_m(\infty;\delta(t)) + (\dot\sigma l/V)C_{m_{\dot\sigma}}(\delta(t)) \\ C_n(t) &= (\dot\lambda l/V)(1/\gamma)C_{n_{\dot\phi}}(\infty;\delta(t)) + (\dot\psi l/V)C_{n_{\dot\psi}}(\infty;\delta(t)) \\ C_l(t) &= 0 \end{aligned} \tag{10}$$

Comparison with Murphy's Formulation

In Refs. 4 and 5, Murphy proposed a nonlinear form for the aerodynamic moment system of a nonrolling body ($p_B = 0$), based on formal expansion in powers of the complex-crossflow velocity vector, its derivative, and conjugates thereof. His result, referred to the nonrolling axis system $\tilde y$, $\tilde z$; reads

$$\tilde C_m + i\tilde C_n = -i\{[c_0 + c_2\delta^2 + c_2{}^*(l/V)(\dot{\delta^2})]\tilde\xi + (d_0 + d_2\delta^2)(l/V)\dot{\tilde\xi}\} \tag{11}$$

where

$$\tilde\xi = (\tilde v + i\tilde w)/V = \delta e^{i\lambda} \tag{12}$$

We may compare this result with Eq. (10) by transferring it to the axis system y, z through the relation

$$C_m + iC_n = ie^{-i\lambda}(\tilde C_m + i\tilde C_n) \tag{13}$$

Substituting Eq. (11) in Eq. (13) and separating into real and imaginary parts yields

$$\begin{aligned} C_m &= \delta(c_0 + c_2\delta^2) + (\dot\delta l/V)(d_0 + d_2\delta^2 + 2c_2{}^*\delta^2) \\ C_n &= \delta(\dot\lambda l/V)(d_0 + d_2\delta^2) \end{aligned} \tag{14}$$

so that, by direct comparison with Eq. (10) (note that with $p_B = 0$ we have $\dot\psi = -\dot\lambda$)

$$\begin{aligned} C_m(\infty;\delta) &= \delta(c_0 + c_2\delta^2) \\ (1/\gamma)C_{m_{\dot\sigma}}(\delta) &= d_0 + d_2\delta^2 + 2c_2{}^*\delta^2 \\ (1/\gamma)C_{n_{\dot\phi}}(\infty;\delta) - C_{n_{\dot\psi}}(\infty;\delta) &= \delta(d_0 + d_2\delta^2) \end{aligned} \tag{15}$$

Thus, our result is compatible with Murphy's form, extends it to arbitrary variations in δ, and assigns a physical meaning to his coefficients.

The form of Murphy's result reemphasizes an important point: To first order in δ (i.e., small δ where a linearized theory can be expected to hold), symmetry requires that $C_{m_{\dot\sigma}}$ and $(1/\delta)(C_{n_{\dot\phi}} - \gamma C_{n_{\dot\psi}})$ be equal; but this requirement does not hold for larger values of δ where terms of $0(\delta^2)$ must be retained. The breakdown of the symmetry requirement has been at least partially confirmed by the results of recent experiments on spinning and coning motions of slender cones at supersonic speeds.[2] These results, obtained with an apparatus that allows investigation of separate or combined coning and spinning motions, are significant in two respects. First, experiments with purely spinning cones at constant δ (i.e., the classical Magnus experiment, $\dot\psi$ = const, $\dot\phi = 0$) failed to reveal a measurable Magnus moment coefficient $C_{n_{\dot\psi}}$; and second, the results confirm that with $C_{n_{\dot\psi}}$ negligibly small, the equality that should exist at small δ is an equality between the side-moment coefficient due to coning $C_{n_{\dot\phi}}/\delta$ and $C_{m_{\dot\sigma}}$. The main results are shown in Fig. 2, where measured values of $C_{n_{\dot\phi}}/\delta$ are compared with the theoretical value of $C_{m_{\dot\sigma}}$. It will be seen that there is excellent agreement between the two at small δ. For larger δ, however, where the leeside vortices appear and become asymmetrically displaced, the equality breaks down. Although a conclusive demonstration of the breakdown of the equality will require a separate experiment for $C_{m_{\dot\sigma}}$ at large σ, we can see no reason why it should be expected to hold when the coefficients become nonlinear functions of σ. Subsequently, we wish to show how failure to account for this behavior in the interpretation of flight data can yield misleading results.

Incorporation of Moment System in Equations of Motion

The moment system proposed here [Eq. (10)] can be incorporated in existing equations of motion programs by simple transformations. Adapted to equations of motion written in body-axis coordinates, the moment system would read

$$C_{m_B} + iC_{n_B} = e^{-i\psi}(C_m + iC_n) \tag{16}$$

whereas, adapted to the equations of motion written in the nonrolling axis system used in aeroballistics, the moment system would read

$$\tilde C_m + i\tilde C_n = -ie^{i\lambda}(C_m + iC_n) \tag{17}$$

In either case, C_m and C_n are given by Eq. (10).

Using Eq. (17) and following Murphy,[4] we can reduce the equations of motion written in nonrolling coordinates to a single complex equation of the following form

$$\tilde\xi'' + [H_2 - (\gamma'/\gamma) - iP]\tilde\xi' - [M + iPT - i\lambda'(H_1 - H_2)]\tilde\xi = G \tag{18}$$

where

$$\tilde{\xi} = \delta e^{i\lambda}$$

$$M = \frac{\rho Sl}{2m}\gamma\left\{\frac{ml^2}{I_{yB}}\frac{C_m(\infty;\delta)}{\delta} - \left[\frac{C_L(\infty;\delta)}{\delta}\right]'\right\}$$

$$H_1 = \frac{\rho Sl}{2m}\left\{\gamma\frac{C_L(\infty;\delta)}{\delta} - C_D(\infty;\delta) - \frac{ml^2}{I_{yB}}\left[\frac{C_{n_{\dot\phi}}(\infty;\delta)}{\delta} - \gamma\frac{C_{n_{\dot\psi}}(\infty;\delta)}{\delta}\right]\right\}$$

$$H_2 = \frac{\rho Sl}{2m}\left[\gamma\frac{C_L(\infty;\delta)}{\delta} - C_D(\infty;\delta) - \frac{ml^2}{I_{yB}}C_{m_{\dot\sigma}}(\delta)\right]$$

$$T = \frac{\rho Sl}{2m}\left\{\gamma\left[\frac{C_L(\infty;\delta)}{\delta} + \frac{ml^2}{I_{xB}}\frac{C_{n_{\dot\psi}}(\infty;\delta)}{\delta}\right] - \frac{I_{yB}}{I_{xB}}\left[\frac{C_{Y_{\dot\psi}}(\infty;\delta)}{\delta}\right]'\right\}$$

$$P = (I_{xB}/I_{yB})p_B l/V,\ s = (1/l)\int V dt$$

and G is the (normally negligible) gravitational contribution. Primes indicate differentiation with respect to s. Alternatively, by use of the equality

$$i\lambda' = \tilde{\xi}'/\tilde{\xi} - \delta'/\delta \tag{19}$$

Equation (18) can be cast in the form

$$\tilde{\xi}'' + (H_1 - (\gamma'/\gamma) - iP)\tilde{\xi}' - [M + iPT + (\delta'/\delta)(H_1 - H_2)]\tilde{\xi} = G \tag{20}$$

Equation (20) is our counterpart of Murphy's result (Eq. (3.28), p. 203, Ref. 4), the latter being obtained when Eq. (11) is used for the aerodynamic moment system.

Discussion

We wish now to examine an important consequence of the failure to account for the breakdown that probably occurs in the equality between $(1/\delta)(C_{n_{\dot\phi}} - \gamma C_{n_{\dot\psi}})$ and $C_{m_{\dot\sigma}}$. We shall show that it may result in an erroneous interpretation of the role played by Magnus moment in determining free-flight motions. Suppose we have available flight data that indicate the existence of a circular limit motion δ = const, λ' = const. When the gravitational contribution can be neglected, Eq. (18) indicates that a necessary condition on the aerodynamic parameters for the presence of a circular limit motion is

$$\lambda' H_1 - PT = 0 \tag{21}$$

It is also necessary that $H_1 + H_2 > 0$. Suppose p_B is such that $\dot\psi' = 0$ (i.e., the body does not spin relative to the cross-flow-velocity vector). Then the classical Magnus moment ought to be identically zero, so that $C_{n_{\dot\psi}}$ ought not contribute to the condition Eq. (21). With $\dot\psi' = 0$, we have $P = (I_{xB}/I_{yB})\lambda'$, so that Eq. (21) becomes

$$H_1 - (I_{xB}/I_{yB})T = 0 \tag{22}$$

Substituting from Eq. (18), and omitting $(C_{Y_{\dot\psi}}/\delta)'$, we have

$$\gamma C_L/\delta[1 - (I_{xB}/I_{yB})] - C_D - (ml^2/I_{yB})C_{n_{\dot\phi}}/\delta = 0 \tag{23}$$

As anticipated, $C_{n_{\dot\psi}}$ does not appear in Eq. (23). Suppose now, however, that the method being used for extracting nonlinear aerodynamic coefficients from the flight data has incorporated within it an assumption that $(1/\delta)(C_{n_{\dot\phi}} - \gamma C_{n_{\dot\psi}})$ equals $C_{m_{\dot\sigma}}$ for all δ. In effect, one has assumed that $H_1 - H_2$ is identically zero for all δ in Eq. (18). Then the condition Eq. (23) becomes

$$\gamma\frac{C_L}{\delta}\left(1 - \frac{I_{xB}}{I_{yB}}\right) - C_D - \frac{ml^2}{I_{yB}}C_{m_{\dot\sigma}} - \frac{ml^2}{I_{yB}}\gamma\frac{C_{n_{\dot\psi}}}{\delta} = 0 \tag{24}$$

The first three terms add up essentially to $2mH_2/\rho Sl$ (for $I_{xB}/I_{yB} \ll 1$), which must be greater than zero. Hence, the only way left for the combination to equal zero is for $C_{n_{\dot\psi}}$ to make up the difference, that is, to be positive and potentially large. This is what the method of extracting coefficients would yield, and one would have to conclude that the Magnus moment is an important factor in determining the motion whereas, in fact, it does not contribute at all.

This result, it is hoped, should serve as a caution to those proposing methods of extracting nonlinear aerodynamic coefficients from flight data. A proposed method will be no better than the form of the aerodynamic moment system incorporated within it.

References

[1] Tobak, M., Schiff, L. B., and Peterson, V. L., "Aerodynamics of Bodies of Revolution in Coning Motion," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 95–99.

[2] Schiff, L. B. and Tobak, M., "Results From a New Wind-Tunnel Apparatus for Studying Coning and Spinning Motions of Bodies of Revolution," *AIAA Journal*, Vol. 8, No. 11, Nov. 1970, pp. 1953–1957.

[3] Kuhn, G. D., Spangler, S. B., and Nielsen, J. N., "Theoretical Study of Vortex Shedding From Bodies of Revolution Undergoing Coning Motion," CR 1448, 1969, NASA.

[4] Murphy, C. H., "Free-Flight Motion of Symmetric Missiles," Rept. 1216, July 1963, Ballistic Research Lab., Aberdeen Proving Ground, Md.

[5] Murphy, C. H., "Circular Pitching and Yawing Motion of Nose Cone Configurations," *Ballistic Missiles and Space Technology*, Vol. 2, Pergamon Press, New York, 1961, pp. 328–336.